

## Designing Fuzzy Backstepping Adaptive Based Fuzzy Estimator Variable Structure Control: Applied to Internal Combustion Engine

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**Abstract:** This paper expands a Multi Input Multi Output (MIMO) fuzzy estimator variable structure control (VSC) which controller coefficient is on-line tuned by fuzzy backstepping algorithm. The main goal is to guarantee acceptable trajectories tracking between the internal combustion engine (IC engine) air to fuel ratio and the desired input. The fuzzy controller in proposed fuzzy estimator variable structure controller is based on Lyapunov fuzzy inference system (FIS) with minimum model based rule base. The input represents the function between variable structure function, error and the rate of error. The outputs represent fuel ratio, respectively. The fuzzy backstepping methodology is on-line tune the variable structure function based on adaptive methodology. The performance of the MIMO fuzzy estimator VSC which controller coefficient is on-line tuned by fuzzy backstepping algorithm (FBAFVSC) is validated through comparison with VSC and proposed method. Simulation results signify good performance of fuel ratio in presence of uncertainty and external disturbance.

**Key words:** internal combustion engine, variable structure controller, fuzzy backstepping controller, chattering phenomenon, adaptive methodology, proposed fuzzy estimator variable structure controller, Lyapunov based controller.

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### **Motivation, Introduction and Background:**

#### **Motivation:**

Internal combustion (IC) engines are optimized to meet exhaust emission requirements with the best fuel economy. Closed loop combustion control is a key technology that is used to optimize the engine combustion process to achieve this goal. In order to conduct research in the area of closed loop combustion control, a control oriented cycle-to-cycle engine model, containing engine combustion information for each individual engine cycle as a function of engine crank angle, is a necessity. Air-to-fuel (A/F) ratio is the mass ratio of air to fuel trapped inside a cylinder before combustion begins, and it affects engine emissions, fuel economy, and other performances. In this research, a fuzzy backstepping adaptive MIMO fuzzy estimator variable structure control scheme is used to simultaneously control the mass flow rate of port fuel injection (PFI) systems to regulate the A/F ratio of PFI to desired levels. One of the most important challenges in the field of IC engine is IC engine control, because this system is MIMO, nonlinear, time variant parameter and uncertainty (Heywood, J., 1988; Ferguson, C., 2001; Guzzella, L., 2004; Ramos, J., 1989; Blair, G., 1999; Zhu, G., *et al*, 2007; Haskara, I., *et al*, 2004; Dawson, J., 2005; Lee, B., 2005). Presently, IC engines are used in different (unknown and/or unstructured) situation consequently caused to provide complicated systems, as a result strong mathematical theory are used in new control methodologies to design nonlinear robust controller. Classical and non-classical methods are two main categories of nonlinear plant control, where the conventional (classical) control theory uses the classical method and the non-classical control theory (e.g., fuzzy logic, neural network, and neuro fuzzy) uses the artificial intelligence methods. However both of conventional and artificial intelligence theories have applied effectively in many areas, but these methods also have some limitations (Kurfess, T.R., 2005; Slotine, J.J.E. and W. Li, 1991).

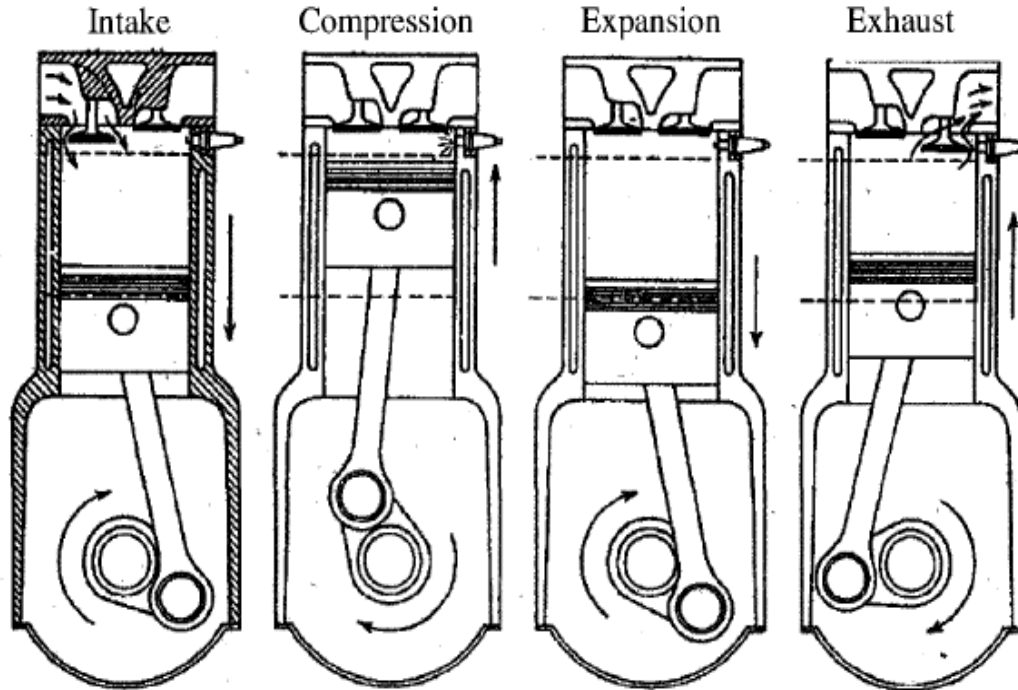
### **INTRODUCTION**

Modeling of an entire IC engine is a very important and complicated process because engines are nonlinear, multi inputs-multi outputs and time variant. One purpose of accurate modeling is to save development costs of real engines and minimizing the risks of damaging an engine when validating controller designs. Nevertheless, developing a small model, for specific controller design purposes, can be done and then validated on a larger, more complicated model (Heywood, J., 1988; Ferguson, C., 2001; Guzzella, L., 2004; Ramos, J., 1989; Blair, G., 1999; Zhu, G., *et al*, 2007; Haskara, I., *et al*, 2004; Dawson, J., 2005; Lee, B., 2005). Dynamic modeling of IC engines is used to describe the behavior of this system, design of model based controller, and for simulation. The dynamic modeling describes the relationship between nonlinear output formulation to electrical or

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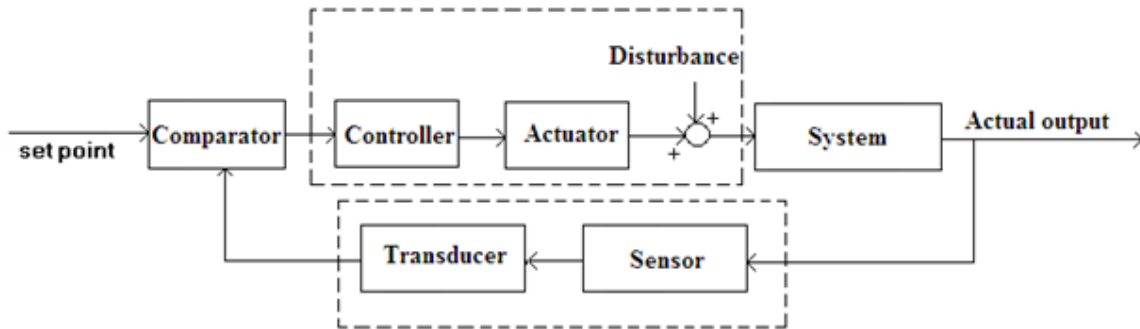
mechanical source and also it can be used to describe the particular dynamic effects to behavior of system (Kurfess, T.R., 2005). In an internal combustion engine, a piston moves up and down in a cylinder and power is transferred through a connecting rod to a crank shaft. The continual motion of the piston and rotation of the crank shaft as air and fuel enter and exit the cylinder through the intake and exhaust valves is known as an engine cycle. The first and most significant engine among all internal combustion engines is the Otto engine, which was developed by Nicolaus A. Otto in 1876 (Figure 1). In his engine, Otto created a unique engine cycle that consisted of four piston strokes. These strokes are: intake stroke, compression stroke, expansion stroke and exhaust stroke (Heywood, J., 1988; Ferguson, C., 2001; Guzzella, L., 2004; Ramos, J., 1989; Blair, G., 1999; Zhu, G., *et al*, 2007; Haskara, I., *et al*, 2004; Dawson, J., 2005; Lee, B., 2005).



**Fig. 1:** The Four Stroke Engine Cycle (Kurfess, T.R., 2005).

Controller (control system) is a device which can sense information from linear or nonlinear system (e.g., IC engine) to improve the systems performance (Piltan, F., *et al.*, 2011; Cheng, L., *et al.*, 2008; D'Azzo, J.J., *et al.*, 2003; Siciliano, B. and O. Khatib, 2008; Boiko, I., *et al.*, 2007; Wang, J. *et al.*, 2001; Farzin Piltan, 2011; Piltan, F., *et al.*, 2011; Piltan, F., *et al.*, 2011; Piltan, F., *et al.*, 2011; Piltan, F., *et al.*, 2011; Piltan, F., *et al.*, 2011; Harashima, F., 1986; Piltan, F., *et al.*, 2011; Utkin, V., 2002; DeCarlo, R.A., *et al.*, 2002; Young, K.D., *et al.*, 2002; Kaynak, O., 2001). There are two control systems: open-loop and close-loop (feedback), by using the open-loop system the control action is independent of the output and in close-loop system the control action depends on the output. In feedback control system considering that there are many disturbances and also variable dynamic parameters something that is really necessary is keeping plant variables close to the desired value. Feedback control system development is the most important thing in many different fields of engineering. The feedback controllers also have classified into two general groups; positive feedback and negative feedback, which in positive the feedback signal can amplify the effect of the input signal and in negative the feedback signal can reduce the effect of the input signal. Figure 2 shows a basic block diagram of a simple negative feedback system with single-input, single-output (SISO). The main targets in designing control systems are stability, good disturbance rejection, and small tracking error (D'Azzo, J.J., *et al.*, 2003; Siciliano, B. and O. Khatib, 2008; Boiko, I., *et al.*, 2007; Wang, J. *et al.*, 2001; Farzin Piltan, 2011; Piltan, F., *et al.*, 2011; Piltan, F., *et al.*, 2011; Piltan, F., *et al.*, 2011; Harashima, F., 1986; Piltan, F., *et al.*, 2011; Utkin, V., 2002; DeCarlo, R.A., *et al.*, 2002; Young, K.D., *et al.*, 2002; Kaynak, O., 2001; Slotine, J.J. and S. Sastry, 1983; Slotine, J.J.E., 1984; Palm, R., 2002; Weng, C.C. and W.S. Yu, 2008; Ertugrul, M. and O. Kaynak, 2000; Kachroo, P. and M. Tomizuka, 2002; Elmali, H. and N. Olgac, 2002; Moura, J. and N. Olgac, 2002; Li, Y. and Q. Xu, 2010). It is a well known fact, the aim of science and modern technology has making an easier life. Conversely, modern life includes complicated technical systems which these systems are nonlinear, time variant and uncertain in measurement, they need to have controlled. Consequently it is hard to design accurate models for these physical systems because they are uncertain. From the control point of view uncertainty is divided into two main groups: uncertainty unstructured inputs (e.g., noise, disturbance) and

uncertainty structure dynamics (e.g., parameter variations). At present, in some applications IC engines are used in unknown and unstructured environment, therefore strong mathematical tools used in new control methodologies to design nonlinear robust controller with an acceptable performance (e.g., minimum error, good trajectory, disturbance rejection). One of the best nonlinear robust controllers is variable structure control which is used in nonlinear uncertain systems.



**Fig. 2:** Block diagram of a feedback control (Piltan, F., *et al.*, 2011).

One of the nonlinear robust controllers is variable structure controller, although this controller has been analyzed by many researchers but the first proposed was in the 1950 (Barrero, F., *et al.*, 2002; Hsu, Y.C. and H.A. Malki, 2002; Hsueh, Y.C., *et al.*, 2009; Shahnazi, R., *et al.*, 2008; Chiang, C.C. and C.H. Wu, 2007; Temeltas, H., 2002; Hwang, C.L. and S.F. Chao, 2005; Lhee, C.G., *et al.*, 2002; Lhee, C.G., 1999; Zhang, X., *et al.*; Emami, M.R., *et al.*, 1998; Lee, H.K., K. Fms, 1994; Kovacic, Z. and S. Bogdan, 2006; Hsu, F.Y. and L.C. Fu, 2002; Berstecher, R.G., *et al.*, 2002; Kim, V., 2002; Wang, Y. and T. Chai, 2005; Yoo, B.K. and W.C. Ham, 2002; Medhaffar, H., *et al.*, 2006; Guo, Y. and P.Y. Woo, 2003; Lin, C.M. and C.F. Hsu, 2004; Iordanov, H.N., B.W. Surgenor, 1997). This controller is used in wide range areas such as in robotics, in control process, in aerospace applications and in IC engines because this methodology can solve some main challenging topics in control such as resistivity to the external disturbance and stability. Even though, this controller is used in wide range areas but, pure variable structure controller has two drawbacks: Firstly, output oscillation (chattering); which caused the heating in the mechanical parameters. Secondly, nonlinear dynamic formulation of nonlinear systems which applied in nonlinear dynamic nonlinear controller; calculate this control formulation is absolutely difficult because it depends on the dynamic nonlinear system's equation (Kaynak, O., 2001; Slotine, J.J. and S. Sastry, 1983; Slotine, J.J.E., 1984; Palm, R., 2002). Chattering phenomenon can causes some problems such as saturation and heats the mechanical parts of IC engine or drivers. To reduce or eliminate the oscillation, various papers have been reported by many researchers which one of the best method is; boundary layer saturation method (Kurfess, T.R., 2005). In boundary layer linear saturation method, the basic idea is the discontinuous method replacement by linear continuous saturation method with small neighborhood of the switching surface. This replacement caused to considerable chattering reduction. Estimated uncertainty method used in term of uncertainty estimator to compensation of the system uncertainties. It has been used to reduce the chattering phenomenon and also solve nonlinear dynamic formulation. This research introduced novel applied MIMO fuzzy inference engine to variable structure controller to estimate the nonlinear control formulation with low computation load. In recent years, artificial intelligence theory has been used in variable structure control systems. Neural network, fuzzy logic, and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant, and uncertainty plant (e.g., IC engines). Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory (Wu, B., *et al.*, 2006; Zadeh, L.A., 1997; Reznik, L., 1997; Zhou, J. and P. Coiffet, 2002; Banerjee, S. and P.Y. Woo, 2002; Kumbala, K., *et al.*, 2000; Lee, C.C. 1990; Wai, R.J., *et al.*, 2003; Wai, R.J. and M.C. Lee, 2004; Menhaj, M.B. and M. Rouhani, 2002; Mohan, S. and S. Bhanot, 2006; Barrero, F., *et al.*, 2002). This controller can be used to control nonlinear, uncertain, and noisy systems. This method is free of some model-based techniques as in classical controllers. As mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems (Wu, B., *et al.*, 2006; Zadeh, L.A., 1997; Reznik, L., 1997; Zhou, J. and P. Coiffet, 2002; Banerjee, S. and P.Y. Woo, 2002; Kumbala, K., *et al.*, 2000) but also this method can help engineers to design easier controller. Control IC engines using classical controllers are based on IC engines dynamic modelling. These controllers often have many problems for modelling. Conventional controllers require accurate information of dynamic model of IC engines, but these models are MIMO, non-linear and calculate accurate dynamic modelling is definitely difficult. When the system model is unknown or when it is known but complicated, it is difficult or impossible to use classical mathematics to process this model (Reznik, L., 1997). The main reasons to use fuzzy logic technology are able to give approximate recommended solution for unclear and complicated systems to easy understanding and flexible. Fuzzy logic provides a method which is able to model a controller for nonlinear plant with a set of IF-THEN

rules, or it can identify the control actions and describe them by using fuzzy rules. It should be mentioned that application of fuzzy logic is not limited to a system that's difficult for modeling, but it can be used in clear systems that have complicated mathematics models because most of the time it can be shortened in design but there is no high quality design just sometimes we can find design with high quality. Besides using fuzzy logic in the main controller of a control loop, it can be used to design adaptive control, tuning parameters, working in a parallel with the classical and non classical control method (Reznik, L., 1997). Research on applied fuzzy logic methodology in variable structure controller (FVSC) to reduce or eliminate the high frequency oscillation (chattering) and to compensate the unknown system dynamics pure variable structure controller considerably improves the nonlinear plant control process (Hsu, Y.C. and H.A. Malki, 2002; Hsueh, Y.C., *et al.*, 2009). Investigation on applied variable structure methodology in fuzzy logic controller (VSFC) to reduce the fuzzy rules and refine the stability of close loop system in fuzzy logic controller has grown specially in recent years as the nonlinear plant control (Palm, R., 2002); (Lhee, C.G., *et al.*, 2002; Lhee, C.G., 1999). However the application of FVSC and VSFC are growing but the main VSFC drawback compared to FVSC is calculation the value of structure surface  $\lambda$  pre-defined very carefully. The advantages of VSFC compared to fuzzy logic controller (FLC) is reduce the number of fuzzy rule base and increase the robustness and stability. At last FVSC compare to the VSFC is more suitable for implementation action. In various dynamic parameters systems that need to be training on-line adaptive control methodology is used. Fuzzy adaptive method is used in systems which want to training parameters by expert knowledge. Traditional adaptive method is used in systems which some dynamic parameters are known. In this research in order to solve disturbance rejection and uncertainty dynamic parameter, adaptive method is applied to artificial variable structure controller.

### **Background:**

Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering (Slotine, J.J. and S. Sastry, 1983). Slotine has presented variable structure method with boundary layer to improve the industry application (Slotine, J.J.E., 1984). R. Palm has presented a fuzzy method to nonlinear approximation instead of linear approximation inside the boundary layer to improve the chattering and control the result performance (Palm, R., 2002). Moreover, C.C. Weng and W.S. Yu improved the previous method by using a new method in fuzzy nonlinear approximation inside the boundary layer and adaptive method (Weng, C.C. and W.S. Yu, 2008). Wu *et al.* (2006) have proposed a simple fuzzy estimator controller beside the discontinuous and equivalent control terms to reduce the chattering. Their design had three main parts i.e. equivalent, discontinuous and fuzzy estimator tuning part which has reduced the chattering very well. Elmali *et al.* (2002) and Li and Xu (2010) have addressed variable structure control with perturbation estimation method (VSCPE) to reduce the classical variable structure chattering. Wai *et al.* (2003, 2004) have proposed a fuzzy neural network (FNN) optimal control system to learn a nonlinear function in the optimal control law. This controller is divided into three main groups: arterial intelligence controller (fuzzy neural network) which it is used to compensate the system's nonlinearity and improves by adaptive method, robust controller to reduce the error and optimal controller which is the main part of this controller. Mohan and Bhanot (2006) have addressed comparative study between some adaptive fuzzy, and a new hybrid fuzzy control algorithm for manipulator control. They found that self-organizing fuzzy logic controller and proposed hybrid integrator fuzzy give the best performance as well as simple structure. H. Temeltas (2002) has proposed fuzzy adaption techniques for VSC to achieve robust tracking of nonlinear systems and solves the chattering problem. Conversely system's performance is better than variable structure controller; it is depended on nonlinear dynamic equation. C.L. Hwang *et al.* (2005) have proposed a Tagaki-Sugeno (TS) fuzzy model based variable structure control based on  $N$  fuzzy based linear state-space to estimate the uncertainties. A MIMO FVSC reduces the chattering phenomenon and reconstructs the approximate the unknown system has been presented for a nonlinear system (Hsu, Y.C. and H.A. Malki, 2002). Yoo and Ham (2002) have proposed a MIMO fuzzy system to help the compensation and estimation the torque coupling. This method can only tune the consequence part of the fuzzy rules. Medhafer *et al.* (2006) have proposed an indirect adaptive fuzzy variable structure controller to control nonlinear system. This MIMO algorithm, applies to estimate the nonlinear dynamic parameters. Compared with the previous algorithm the numbers of fuzzy rules have reduced by introducing the variable structure surface as inputs of fuzzy systems. Y. Guo and P.Y. Woo (2003) have proposed a SISO fuzzy system compensate and reduce the chattering. C. M. Lin and C. F. Hsu [61] can tune both systems by fuzzy rules. Shahnazi *et al.*, have proposed a SISO PI direct adaptive fuzzy variable structure controller based on Lin and Hsu algorithm to reduce or eliminate chattering. The bounds of PI controller and the parameters are online adjusted by low adaption computation (Shahnazi, R., *et al.*, 2008).

In this research we will highlight the MIMO adaptive backstepping fuzzy variable structure algorithm with estimates the nonlinear dynamic part derived in the Lyapunov sense. This algorithm will be analyzed and evaluated on IC engine. Section 2, serves as an introduction to the variable structure formulation algorithm and its application to an IC engine, describe the objectives and problem statements. Part 3, introduces and describes

the methodology algorithms and proves Lyapunov stability. Section 4 presents the simulation results of this algorithm applied to an IC engine and the final section is describe the conclusion.

**Objectives, Problem Statements and Variable Structure Formulation Applied to Ic Engine: Dynamic of IC Engine:**

In developing a valid engine model, the concept of the combustion process, abnormal combustion, and cylinder pressure must be understood. The combustion process is relatively simple and it begins with fuel and air being mixed together in the intake manifold and cylinder. This air-fuel mixture is trapped inside cylinder after the intake valve(s) is closed and then gets compressed (Piltan, F., *et al.*, 2011). When the air-fuel mixture is compressed it causes the pressure and temperature to increase inside the cylinder. Unlike normal combustion, the cylinder pressure and temperature can rise so rapidly that it can spontaneously ignite the air-fuel mixture causing high frequency cylinder pressure oscillations. These oscillations cause the metal cylinders to produce sharp noises called knock, which it caused to abnormal combustion. The pressure in the cylinder is a very important physical parameter that can be analyzed from the combustion process. Since cylinder pressure is very important to the combustion event and the engine cycle in spark ignition engines, the development of a model that produces the cylinder pressure for each crank angle degree is necessary. A cylinder pressure model that calculates the total cylinder pressure over 720 crank angle degrees was created based upon the following formulation (Heywood, J., 1988; Ferguson, C., 2001; Guzzella, L., 2004; Ramos, J., 1989; Blair, G., 1999; Zhu, G., *et al.*, 2007; Haskara, I., *et al.*, 2004; Dawson, J., 2005; Lee, B., 2005):

$$P_{cyl}(\theta) = P_m(\theta) + P_{net}(\theta) \tag{1}$$

where  $P_{cyl}(\theta)$  is pressure in cylinder,  $P_m(\theta)$  is Wiebe function, and  $P_{net}(\theta)$  is motoring pressure of a cylinder. Air fuel ratio is the mass ratio of air and fuel trapped inside the cylinder before combustion starts. Mathematically it is the mass of the air divided by the mass of the fuel as shown in the equation below:

$$Air\ to\ Fuel = \frac{\dot{m}_{air}}{\dot{m}_{fuel}} \tag{2}$$

If the ratio is too high or too low, it can be adjusted by adding or reducing the amount of fuel per engine cycle that is injected into the cylinder. The fuel ratio can be used to determine which fuel system should have a larger impact on how much fuel is injected into the cylinder. Since a direct fuel injector has immediate injection of its fuel with significant charge cooling effect, it can have a quicker response to the desired amount of fuel that is needed by an engine (Ramos, J., 1989).

**Variable Structure Methodology:**

Based on variable structure discussion, the control law for a multi degrees of freedom robot manipulator is written as (DeCarlo, R.A., *et al.*, 2002; Young, K.D., *et al.*, 2002; Kaynak, O., 2001; Slotine, J.J. and S. Sastry, 1983; Slotine, J.J.E., 1984; Palm, R., 2002; Weng, C.C. and W.S. Yu, 2008):

$$U = U_{Nonlinear} + U_{dis} \tag{9}$$

Where, the model-based component  $U_{Nonlinear}$  is compensated the nominal dynamics of systems. Therefore  $U_{Nonlinear}$  can calculate as follows:

$$U_{Nonlinear} = [M^{-1}(P_m(\theta) + P_{net}(\theta)) + \dot{S}]M \tag{10}$$

Where

$$M^{-1} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \quad M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

A simple solution to get the variable structure condition when the dynamic parameters have uncertainty is the switching control law:

$$U_{dis} = K(\vec{x}, t) \cdot \text{sgn}(s) \tag{11}$$

where the switching function  $\text{sgn}(S)$  is defined as

$$\mathit{sgn}(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \quad (12)$$

and the  $K(\vec{x}, t)$  is the positive constant.  
the lyapunov formulation can be written as follows,

$$V = \frac{1}{2} S^T \cdot M \cdot S \quad (13)$$

the derivation of  $V$  can be determined as,

$$\dot{V} = \frac{1}{2} S^T \cdot \dot{M} \cdot S + S^T M \dot{S} \quad (14)$$

the dynamic equation of IC engine can be written based on the structure variable surface as

$$M \dot{S} = -VS + M \dot{S} + P_m(\theta) + P_{net}(\theta) \quad (15)$$

it is assumed that

$$S^T (\dot{M} - 2P_m(\theta) + P_{net}(\theta)) S = 0 \quad (16)$$

by substituting (15) in (14)

$$\dot{V} = \frac{1}{2} S^T \dot{M} S - S^T P_m(\theta) + P_{net}(\theta) S + S^T (M \dot{S} + P_m(\theta) + P_{net}(\theta) S) = S^T (M \dot{S} + P_m(\theta) + P_{net}(\theta) S) \quad (17)$$

suppose the control input is written as follows

$$\hat{U} = U_{Nonlinear} + \hat{U}_{dis} = [\hat{M}^{-1}(P_m(\theta) + P_{net}(\theta)) + \dot{S}] \hat{M} + K \cdot \mathit{sgn}(S) + P_m(\theta) + P_{net}(\theta) S \quad (18)$$

by replacing the equation (18) in (17)

$$\dot{V} = S^T (M \dot{S} + P_m(\theta) + P_{net}(\theta) - \hat{M} \dot{S} - P_m(\theta) + \hat{P}_{net}(\theta) S - K \mathit{sgn}(S)) = S^T (\tilde{M} \dot{S} + P_m(\theta) + \hat{P}_{net}(\theta) S - K \mathit{sgn}(S)) \quad (19)$$

it is obvious that

$$|\tilde{M} \dot{S} + P_m(\theta) + \hat{P}_{net}(\theta) S| \leq |\tilde{M} \dot{S}| + |P_m(\theta) + \hat{P}_{net}(\theta) S| \quad (20)$$

the Lemma equation in IC engine system can be written as follows

$$K_u = [|\tilde{M} \dot{S}| + |P_m(\theta) + \hat{P}_{net}(\theta) S| + \eta]_i, i = 1, 2, 3, 4, \dots \quad (21)$$

the equation (16) can be written as

$$K_u \geq [|\tilde{M} \dot{S} + P_m(\theta) + \hat{P}_{net}(\theta) S|]_i + \eta_i \quad (22)$$

therefore, it can be shown that

$$\dot{V} \leq -\sum_{i=1}^n \eta_i |S_i| \quad (23)$$

Consequently the equation (23) guaranties the stability of the Lyapunov equation. Figure 3 is shown the integration of mean value engine and cylinder pressure model.

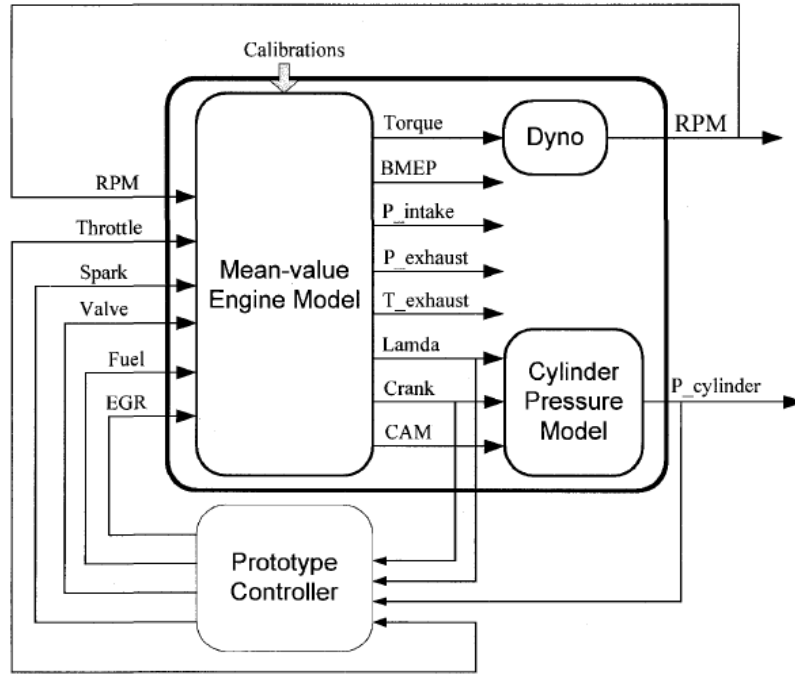


Fig. 3: Block diagram of an integration of mean value engine and cylinder pressure model.

Figure 4 is shown pure variable structure controller, applied to IC engine.

**Problem Statements:**

Even though, variable structure controller is used in wide range areas but, pure it also has chattering problem and nonlinear dynamic part challenges. On the other hand, fuzzy logic controller has been used for nonlinear and uncertain (e.g., IC engine) systems controlling. Conversely pure FLC works in many areas, it cannot guarantee the basic requirement of stability and acceptable performance (Wang, J. *et al.*, 2001). Although both VSC and FLC have been applied successfully in many applications but they also have some limitations. The boundary layer method is used to reduce or eliminate the chattering and proposed fuzzy Lyapunov estimator method focuses on substitution fuzzy logic system instead of dynamic nonlinear equation to implement easily and avoid mathematical model base controller. To reduce the effect of uncertainty in proposed method, MIMO novel adaptive method is applied in fuzzy variable structure controller in IC engine.

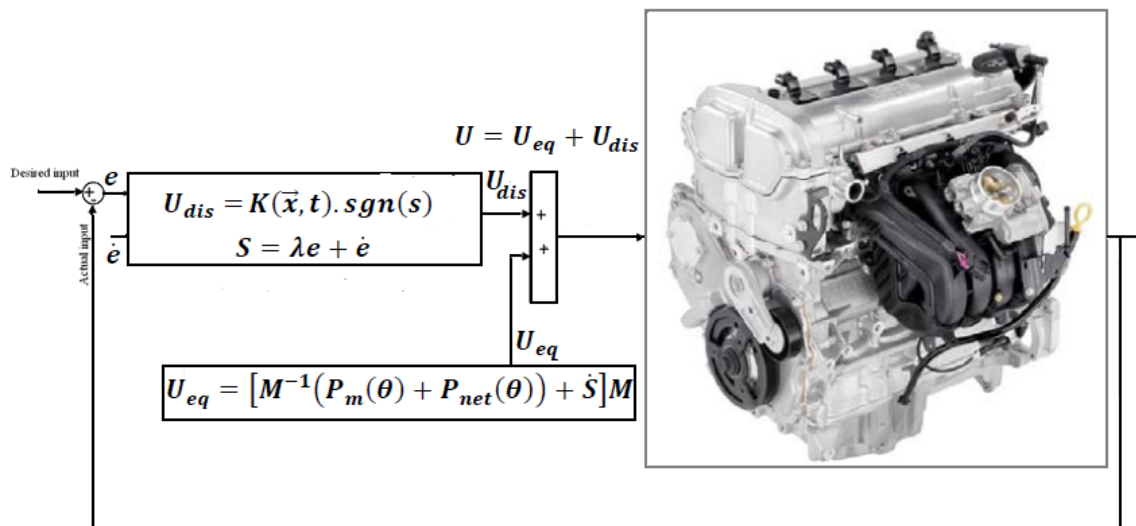


Fig. 4: Block diagram of a variable structure controller: applied to IC engine.

**Objectives:**

The main goal is to design a MIMO fuzzy backstepping adaptive fuzzy estimation variable structure methodology which applied to internal combustion engine with easy to design and implement. IC engine has

nonlinear dynamic and uncertain parameters consequently; following objectives have been pursuit in the mentioned research: To develop a chattering in a position pure variable structure controller against uncertainties, to design and implement a Lyapunov MIMO fuzzy structure variable controller in order to solve the equivalent problems with minimum rule base and finally to develop a position fuzzy backstepping adaptive fuzzy estimation variable structure controller in order to solve the disturbance rejection and reduce the computation load.

**Methodology: Design a Novel Mimo Fuzzy Backstepping Adaptive Fuzzy Estimation Variable Structure Control:**

**First part** is focused on eliminate the oscillation (chattering) in pure variable structure controller based on linear boundary layer method. To reduce or eliminate the chattering it is used the boundary layer method; in boundary layer method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface. This replace is caused to increase the error performance (Kaynak, O., 2001; Slotine, J.J. and S. Sastry, 1983; Slotine, J.J.E., 1984; Palm, R., 2002; Weng, C.C. and W.S. Yu, 2008).

$$B(t) = \{x, |S(t)| \leq \phi\}; \phi > 0 \tag{24}$$

Where  $\phi$  is the boundary layer thickness. Therefore, to have a smote control law, the saturation function  $Sat(S/\phi)$  added to the control law:

$$U = K(\vec{x}, t). Sat(S/\phi) \tag{25}$$

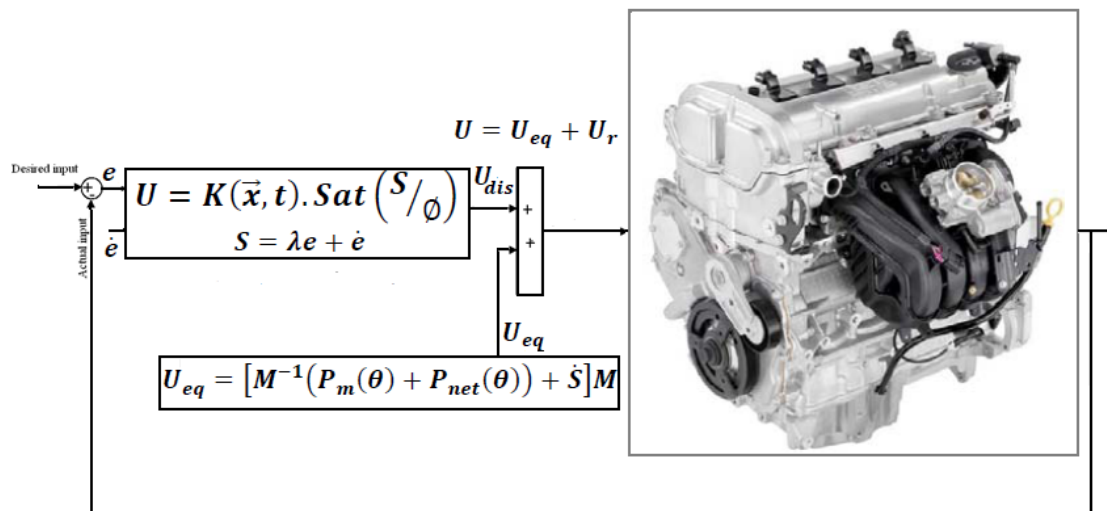
Where  $Sat(S/\phi)$  can be defined as

$$sat(S/\phi) = \begin{cases} 1 & (S/\phi > 1) \\ -1 & (S/\phi < -1) \\ S/\phi & (-1 < S/\phi < 1) \end{cases} \tag{26}$$

Based on above discussion, the control law for an IC engine is written as (DeCarlo, R.A., et al., 2002; Young, K.D., et al., 2002; Kaynak, O., 2001; Slotine, J.J. and S. Sastry, 1983; Slotine, J.J.E., 1984; Palm, R., 2002; Weng, C.C. and W.S. Yu, 2008):

$$U = U_{eq} + U_r \tag{27}$$

Figure 5 is shown classical variable structure which eliminates the chattering using linear boundary layer method.



**Fig. 5:** Chattering free Block diagram of a variable structure controller: applied to IC engine.

**Second step** is focused on design MIMO fuzzy estimation variable structure based on Lyapunov formulation. The first type of fuzzy systems is given by

$$f(x) = \sum_{l=1}^M \theta^l \varepsilon^l(x) = \theta^T \varepsilon(x) \tag{28}$$

Where  $\theta = (\theta^1, \dots, \theta^M)^T$ ,  $\varepsilon(x) = (\varepsilon^1(x), \dots, \varepsilon^M(x))^T$ , and  $\varepsilon^l(x) =: \frac{\mu_{A_i^l}(x_i)}{\sum_{l=1}^M (\prod_{i=1}^n \mu_{A_i^l}(x_i))}$ .  $\theta^1, \dots, \theta^M$  are adjustable parameters in (28).  $\mu_{A_1^1}(x_1), \dots, \mu_{A_n^m}(x_n)$  are given membership functions whose parameters will not change over time.

The second type of fuzzy systems is given by

$$f(x) = \frac{\sum_{l=1}^M \theta^l \left[ \prod_{i=1}^n \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right) \right]}{\sum_{l=1}^M \left[ \prod_{i=1}^n \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right) \right]} \tag{29}$$

Where  $\theta^l$ ,  $\alpha_i^l$  and  $\delta_i^l$  are all adjustable parameters. From the universal approximation theorem, we know that we can find a fuzzy system to estimate any continuous function. For the first type of fuzzy systems, we can only adjust  $\theta^l$  in (28). We define  $f^\wedge(x|\theta)$  as the approximator of the real function  $f(x)$ .

$$f^\wedge(x|\theta) = \theta^T \varepsilon(x) \tag{30}$$

We define  $\theta^*$  as the values for the minimum error:

$$\theta^* = \arg \min_{\theta \in \Omega} [\sup_{x \in U} |f^\wedge(x|\theta) - g(x)|] \tag{31}$$

Where  $\Omega$  is a constraint set for  $\theta$ . For specific  $x$ ,  $\sup_{x \in U} |f^\wedge(x|\theta^*) - f(x)|$  is the minimum approximation error we can get.

We used the first type of fuzzy systems (28) to estimate the nonlinear system (10) the fuzzy formulation can be write as below;

$$f(x|\theta) = \theta^T \varepsilon(x) = \frac{\sum_{l=1}^n \theta^l [\mu_{A^l}(x)]}{\sum_{l=1}^n [\mu_{A^l}(x)]} \tag{32}$$

Where  $\theta^1, \dots, \theta^n$  are adjusted by an adaptation law. The adaptation law is designed to minimize the parameter errors of  $\theta - \theta^*$ . A MIMO (multi-input multi-output) fuzzy system is designed to compensate the uncertainties of the nonlinear system. The parameters of the fuzzy system are adjusted by adaptation laws. The tracking error and the sliding surface state are defined as:

$$e = q - q_d \tag{33}$$

$$s = \dot{e} + \lambda_e \tag{34}$$

We define the reference state as

$$\dot{q}_r = \dot{q} - s = \dot{q}_d - \lambda e \tag{35}$$

$$\ddot{q}_r = \ddot{q} - \dot{s} = \ddot{q}_d - \lambda \dot{e} \tag{36}$$

The general MIMO if-then rules are given by

$$R^l: \text{if } x_1 \text{ is } A_1^l, x_2 \text{ is } A_2^l, \dots, x_n \text{ is } A_n^l, \text{ then } y_1 \text{ is } B_1^l, \dots, y_m \text{ is } B_m^l \tag{37}$$

Where  $l = 1, 2, \dots, M$  are fuzzy if-then rules;  $x = (x_1, \dots, x_n)^T$  and  $y = (y_1, \dots, y_n)^T$  are the input and output vectors of the fuzzy system. The MIMO fuzzy system is define as

$$f(x) = \Theta^T \varepsilon(x) \tag{38}$$

Where

$$\Theta^T = (\theta_1, \dots, \theta_m)^T = \begin{bmatrix} \theta_1^1, \theta_1^2, \dots, \theta_1^M \\ \theta_2^1, \theta_2^2, \dots, \theta_2^M \\ \vdots \\ \theta_m^1, \theta_m^2, \dots, \theta_m^M \end{bmatrix} \quad (39)$$

$\varepsilon(x) = (\varepsilon^1(x), \dots, \varepsilon^M(x))^T$ ,  $\varepsilon^1(x) = \prod_{i=1}^n \mu_{A_i^1}(x_i) / \sum_{i=1}^M (\prod_{i=1}^n \mu_{A_i^i}(x_i))$ , and  $\mu_{A_i^i}(x_i)$  is defined in (32). To reduce the number of fuzzy rules, we divide the fuzzy system in to three parts:

$$\begin{aligned} F^1(q, \dot{q}) &= \Theta^{1T} \varepsilon(q, \dot{q}) \\ &= [\theta_1^{1T} \varepsilon(q, \dot{q}), \dots, \theta_m^{1T} \varepsilon(q, \dot{q})]^T \end{aligned} \quad (40)$$

$$\begin{aligned} F^2(q, \ddot{q}_r) &= \Theta^{2T} \varepsilon(q, \ddot{q}_r) \\ &= [\theta_1^{2T} \varepsilon(q, \ddot{q}_r), \dots, \theta_m^{2T} \varepsilon(q, \ddot{q}_r)]^T \end{aligned} \quad (41)$$

$$\begin{aligned} F^3(q, \ddot{q}) &= \Theta^{3T} \varepsilon(q, \ddot{q}) \\ &= [\theta_1^{3T} \varepsilon(q, \ddot{q}), \dots, \theta_m^{3T} \varepsilon(q, \ddot{q})]^T \end{aligned} \quad (42)$$

The control input is given by

$$\tau = M^{\wedge} \ddot{q}_r + P_m(\theta) + P_{net}(\theta) + F^1(q, \dot{q}) + F^2(q, \ddot{q}_r) + F^3(q, \ddot{q}) - K_D s - W \text{sgn}(s) \quad (43)$$

Where  $M^{\wedge}$ ,  $P_m(\theta) + P_{net}(\theta)$  are the estimations of  $M(q)$  and are positive constants;  $W = \text{diag}[W_1, \dots, W_m]$  and  $W_1, \dots, W_m$  are positive constants. The adaptation law is given by

$$\begin{aligned} \dot{\theta}_j^1 &= -\Gamma_{1j} s_j \varepsilon(q, \dot{q}) \\ \dot{\theta}_j^2 &= -\Gamma_{2j} s_j \varepsilon(q, \ddot{q}_r) \\ \dot{\theta}_j^3 &= -\Gamma_{3j} s_j \varepsilon(q, \ddot{q}) \end{aligned} \quad (44)$$

Where  $j = 1, \dots, m$  and  $\Gamma_{1j} - \Gamma_{3j}$  are positive diagonal matrices.

The Lyapunov function candidate is presented as

$$V = \frac{1}{2} s^T M s + \frac{1}{2} \sum_{j=1}^m \frac{1}{\Gamma_{1j}} \phi_j^{1T} \phi_j^1 + \frac{1}{2} \sum_{j=1}^m \frac{1}{\Gamma_{2j}} \phi_j^{2T} \phi_j^2 + \frac{1}{2} \sum_{j=1}^m \frac{1}{\Gamma_{3j}} \phi_j^{3T} \phi_j^3 \quad (45)$$

Where  $\phi_j^1 = \phi_j^{1*} - \theta_j^1$ ,  $\phi_j^2 = \phi_j^{2*} - \theta_j^2$  and  $\phi_j^3 = \phi_j^{3*} - \theta_j^3$  we define

$$F(q, \dot{q}, \ddot{q}_r, \ddot{q}) = F^1(q, \dot{q}) + F^2(q, \ddot{q}_r) + F^3(q, \ddot{q}) \quad (46)$$

From (31) and (32), we get

$$M(q) \ddot{q} + P_m(\theta) + P_{net}(\theta) = M^{\wedge} \ddot{q}_r + \widehat{P_m(\theta) + P_{net}(\theta)} + F(q, \dot{q}, \ddot{q}_r, \ddot{q}) - K_D s - W \text{sgn}(s) \quad (47)$$

Since  $\dot{q}_r = \dot{q} - s$  and  $\ddot{q}_r = \ddot{q} - \dot{s}$ , we get

$$M \dot{s} + (P_m(\theta) + P_{net}(\theta) + K_D) s + W \text{sgn}(s) = -\Delta F + F(q, \dot{q}, \ddot{q}_r, \ddot{q}) \quad (48)$$

Then  $M \dot{s} + P_m(\theta) + P_{net}(\theta) s$  can be written as

$$M \dot{s} + P_m(\theta) + P_{net}(\theta) s = -\Delta F + F(q, \dot{q}, \ddot{q}_r, \ddot{q}) - K_D s - W \text{sgn}(s) \quad (49)$$

Where  $\Delta F = \tilde{M} \ddot{q}_r + P_m(\theta) + P_{net}(\theta)$ ,  $\tilde{M} = M - M^{\wedge}$ ,  $\tilde{C}_1 = P_m(\theta) + P_{net}(\theta) - \widehat{P_m(\theta) + P_{net}(\theta)}$  The derivative of  $V$  is

$$\dot{V} = s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s + \sum_{j=1}^m \frac{1}{\Gamma_{1j}} \phi_j^{1T} \dot{\phi}_j^1 + \sum_{j=1}^m \frac{1}{\Gamma_{2j}} \phi_j^{2T} \dot{\phi}_j^2 + \sum_{j=1}^m \frac{1}{\Gamma_{3j}} \phi_j^{13T} \dot{\phi}_j^3 \quad (50)$$

We know that  $s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s = s^T (M \dot{s} + P_m(\theta) + P_{net}(\theta) s)$  from (50). Then

$$\dot{V} = -s^T [-K_D s + W s \operatorname{sgn}(s) + \Delta F - F(q, \dot{q}, \ddot{q}_r, \ddot{q})] + \sum_{j=1}^m \frac{1}{\Gamma_{1j}} \phi_j^{1T} \dot{\phi}_j^1 + \sum_{j=1}^m \frac{1}{\Gamma_{2j}} \phi_j^{2T} \dot{\phi}_j^2 + \sum_{j=1}^m \frac{1}{\Gamma_{3j}} \phi_j^{13T} \dot{\phi}_j^3 \quad (51)$$

We define the minimum approximation error as

$$\omega = \Delta F - [F^1(q, \dot{q} | \Theta^{1*}) + F^2(q, \ddot{q}_r | \Theta^{2*}) + F^3(q, \ddot{q} | \Theta^{3*})] \quad (52)$$

We plug (51) in to (52)

$$\begin{aligned} \dot{V} &= -s^T [-K_D s + W s \operatorname{sgn}(s) + \Delta F - F(q, \dot{q}, \ddot{q}_r, \ddot{q})] + \sum_{j=1}^m \frac{1}{\Gamma_{1j}} \phi_j^{1T} \dot{\phi}_j^1 + \sum_{j=1}^m \frac{1}{\Gamma_{2j}} \phi_j^{2T} \dot{\phi}_j^2 + \sum_{j=1}^m \frac{1}{\Gamma_{3j}} \phi_j^{13T} \dot{\phi}_j^3 \\ &= -s^T [-K_D s + W s \operatorname{sgn}(s) + \omega + F^1(q, \dot{q} | \Theta^{1*}) + F^2(q, \ddot{q}_r | \Theta^{2*}) + F^3(q, \ddot{q} | \Theta^{3*}) - F^1(q, \dot{q}) + \\ &F^2(q, \ddot{q}_r) + F^3(q, \ddot{q})] + \sum_{j=1}^m \frac{1}{\Gamma_{1j}} \phi_j^{1T} \dot{\phi}_j^1 + \sum_{j=1}^m \frac{1}{\Gamma_{2j}} \phi_j^{2T} \dot{\phi}_j^2 + \sum_{j=1}^m \frac{1}{\Gamma_{3j}} \phi_j^{13T} \dot{\phi}_j^3 \\ &= -s^T K_D s - s^T W s \operatorname{sgn}(s) - s^T \omega - \sum_{j=1}^m s_j \phi_j^{1T} \varepsilon(q, \dot{q}) - \sum_{j=1}^m s_j \phi_j^{2T} \varepsilon(q, \ddot{q}_r) - \sum_{j=1}^m s_j \phi_j^{3T} \varepsilon(q, \ddot{q}) + \\ &\sum_{j=1}^m \frac{1}{\Gamma_{1j}} \phi_j^{1T} \dot{\phi}_j^1 + \sum_{j=1}^m \frac{1}{\Gamma_{2j}} \phi_j^{2T} \dot{\phi}_j^2 + \sum_{j=1}^m \frac{1}{\Gamma_{3j}} \phi_j^{3T} \dot{\phi}_j^3 \\ &= -s^T K_D s - s^T W s \operatorname{sgn}(s) - s^T \omega - \sum_{j=1}^m \phi_j^{1T} (s_j \varepsilon(q, \dot{q}) - \frac{1}{\Gamma_{1j}} \dot{\phi}_j^1) - \sum_{j=1}^m \phi_j^{2T} (s_j \varepsilon(q, \ddot{q}_r) - \frac{1}{\Gamma_{2j}} \dot{\phi}_j^2) - \\ &\sum_{j=1}^m \phi_j^{3T} (s_j \varepsilon(q, \ddot{q}) - \frac{1}{\Gamma_{3j}} \dot{\phi}_j^3) \\ &= -s^T K_D s - s^T W s \operatorname{sgn}(s) - s^T \omega - \sum_{j=1}^m \phi_j^{1T} (s_j \varepsilon(q, \dot{q}) + \frac{1}{\Gamma_{1j}} \dot{\phi}_j^1) - \sum_{j=1}^m \phi_j^{2T} (s_j \varepsilon(q, \ddot{q}_r) + \\ &\frac{1}{\Gamma_{2j}} \dot{\phi}_j^2) - \sum_{j=1}^m \phi_j^{3T} (s_j \varepsilon(q, \ddot{q}) + \frac{1}{\Gamma_{3j}} \dot{\phi}_j^3) \end{aligned}$$

Then  $\dot{V}$  becomes

$$\begin{aligned} \dot{V} &= -s^T K_D s - s^T W s \operatorname{sgn}(s) - s^T \omega \\ &= - \sum_{j=1}^m (s_j^2 K_{Dj} + W_j |s_j| + s_j \omega_j) \\ &= - \sum_{j=1}^m [s_j (s_j K_{Dj} + \omega_j) + W_j |s_j|] \quad (53) \end{aligned}$$

Since  $\omega_j$  can be as small as possible, we can find  $K_{Dj}$  that  $|s_j^2 K_{Dj}| > |\omega_j| (s_j \neq 0)$ .

Therefore, we can get  $s_j (s_j K_{Dj} + \omega_j) > 0$  for  $s_j \neq 0$  and  $\dot{V} < 0 (s \neq 0)$ . Figure 6 is shown the fuzzy estimator variable structure.

Third step is focused on design Mamdani's fuzzy (Wu, B., *et al.*, 2006; Zadeh, L.A., 1997; Reznik, L., 1997; Zhou, J. and P. Coiffet, 2002; Banerjee, S. and P.Y. Woo, 2002; Kumbala, K., *et al.*, 2000; Lee, C.C. 1990; Wai, R.J., *et al.*, 2003; Wai, R.J. and M.C. Lee, 2004; Menhaj, M.B. and M. Rouhani, 2002; Mohan, S. and S. Bhanot, 2006) backstepping adaptive fuzzy estimator variable structure. As mentioned above pure variable structure controller has nonlinear dynamic equivalent limitations in presence of uncertainty and external disturbances in order to solve these challenges this work applied Mamdani's fuzzy inference engine estimator in variable structure controller. However proposed MIMO fuzzy estimator variable structure has satisfactory performance but calculate the variable structure surface slope by try and error or experience knowledge is very difficult, particularly when system has structure or unstructured uncertainties; MIMO Mamdani's fuzzy backstepping variable structure function fuzzy estimator variable structure controller is recommended. The backstepping method is based on mathematical formulation which this method is introduced new variables into it in form depending on the dynamic equation of IC engine. This method is used as feedback linearization in order to solve nonlinearities in the system. To use of nonlinear fuzzy filter this method in this research makes it possible to create dynamic nonlinear backstepping estimator into the adaptive fuzzy estimator variable structure process to eliminate or reduce the challenge of uncertainty in this part. The backstepping controller is calculated by;

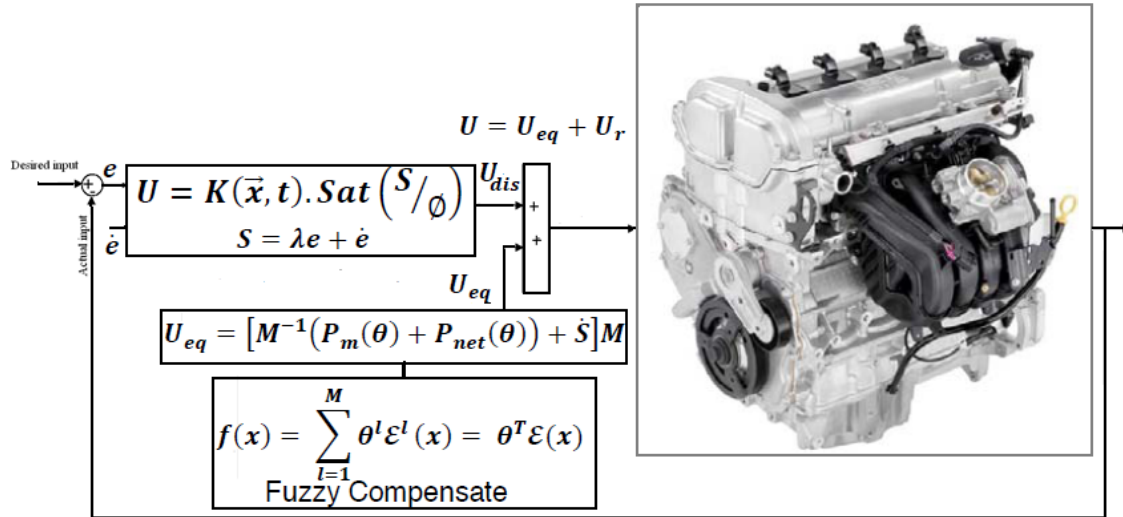


Fig. 6: Chattering free Block diagram of a fuzzy estimator variable structure controller.

$$U_{B,S} = U_{eq_{B,S}} + M \cdot I \tag{54}$$

Where  $U_{B,S}$  is backstepping output function,  $U_{eq_{B,S}}$  is backstepping nonlinear equivalent function which can be written as (55) and  $I$  is backstepping control law which calculated by (56)

$$U_{eq_{B,S}} = [(P_m(\theta) + P_{net}(\theta))] \tag{55}$$

$$I = [\ddot{\theta} + K_1(K_1 - 1) \cdot e + (K_1 + K_2) \cdot \dot{e}] \tag{56}$$

Based on (10) and (55) the fuzzy backstepping filter is considered as

$$(P_m(\theta) + P_{net}(\theta)) = \sum_{l=1}^M \theta^l \zeta(x) - \lambda S - K \tag{57}$$

Based on (54) the formulation of fuzzy backstepping filter can be written as;

$$U = U_{eq_{B,Sfuzzy}} + MI \tag{58}$$

Where  $U_{eq_{B,Sfuzzy}} = [(P_m(\theta) + P_{net}(\theta))] + \sum_{l=1}^M \theta^l \zeta(x) + K$

The adaption low is defined as

$$\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j) \tag{59}$$

where the  $\gamma_{sj}$  is the positive constant and  $\zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \dots, \zeta_j^M(S_j)]^T$

$$\zeta_j^1(S_j) = \frac{\mu_{(A)_j^1}(S_j)}{\sum_i \mu_{(A)_j^i}(S_j)} \tag{60}$$

The dynamic equation of IC engine can be written based on the variable structure surface as;

$$M\dot{S} = -VS + M\dot{S} + VS \tag{61}$$

It is supposed that

$$S^T(\dot{M} - 2V)S = 0 \tag{62}$$

The derivation of Lyapunov function ( $\dot{V}$ ) is written as

$$\begin{aligned}
 \dot{V} &= \frac{1}{2} S^T \dot{M} S - S^T V S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\
 &= S^T (-\lambda S + \Delta f - K) + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\
 &= \sum_{j=1}^m [S_j (\Delta f_j - K_j)] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\
 &= \sum_{j=1}^m [S_j (\Delta f_j - \theta_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\
 &= \sum_{j=1}^m [S_j (\Delta f_j - (\theta_j^*)^T \zeta_j(S_j) + \phi_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\
 &= \sum_{j=1}^m [S_j (\Delta f_j - ((\theta_j^*)^T \zeta_j(S_j)))] - S^T \lambda S + \sum (\frac{1}{\gamma_{sj}} \phi_j^T [\gamma_{sj} \zeta_j(S_j) S_j + \dot{\phi}_j])
 \end{aligned}$$

Where  $\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j)$  is adaption law and  $\dot{\phi}_j = -\dot{\theta}_j = -\gamma_{sj} S_j \zeta_j(S_j)$ , consequently  $\dot{V}$  can be considered by

$$\dot{V} = \sum_{j=1}^m [S_j \Delta f_j - ((\theta_j^*)^T \zeta_j(S_j))] - S^T \lambda S \tag{63}$$

The minimum error can be defined by

$$e_{mj} = \Delta f_j - ((\theta_j^*)^T \zeta_j(S_j)) \tag{64}$$

$\dot{V}$  is intended as follows

$$\begin{aligned}
 \dot{V} &= \sum_{j=1}^m [S_j e_{mj}] - S^T \lambda S \\
 &\leq \sum_{j=1}^m |S_j| |e_{mj}| - S^T \lambda S \\
 &= \sum_{j=1}^m |S_j| |e_{mj}| - \lambda_j S_j^2 \\
 &= \sum_{j=1}^m |S_j| (|e_{mj}| - \lambda_j S_j)
 \end{aligned} \tag{65}$$

For continuous function  $U_{eqB.Sfuzzy}$  and suppose  $\epsilon > 0$  it is defined the fuzzy backstepping controller in form of (58) such that

$$Sup_{x \in U} |U_{eqB.Sfuzzy} + MI| < \epsilon \tag{66}$$

As a result MIMO fuzzy backstepping adaptive fuzzy estimation variable structure is very stable which it is one of the most important challenges to design a controller with suitable response. Figure 7 is shown the block diagram of proposed MIMO fuzzy backstepping adaptive fuzzy estimation variable structure.

**Results:**

Variable structure controller (VSC) and proposed MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller were tested to sinus response trajectory. The simulation was implemented in Matlab/Simulink environment. Fuel ratio trajectory, disturbance rejection and error are compared in these controllers. It is noted that, these systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude. This type of noise is used to external disturbance in continuous and hybrid systems.

**Fuel Ratio Trajectory:**

Figure 8 shows the fuel ratio in VSC and proposed MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller without disturbance for sinus trajectory.

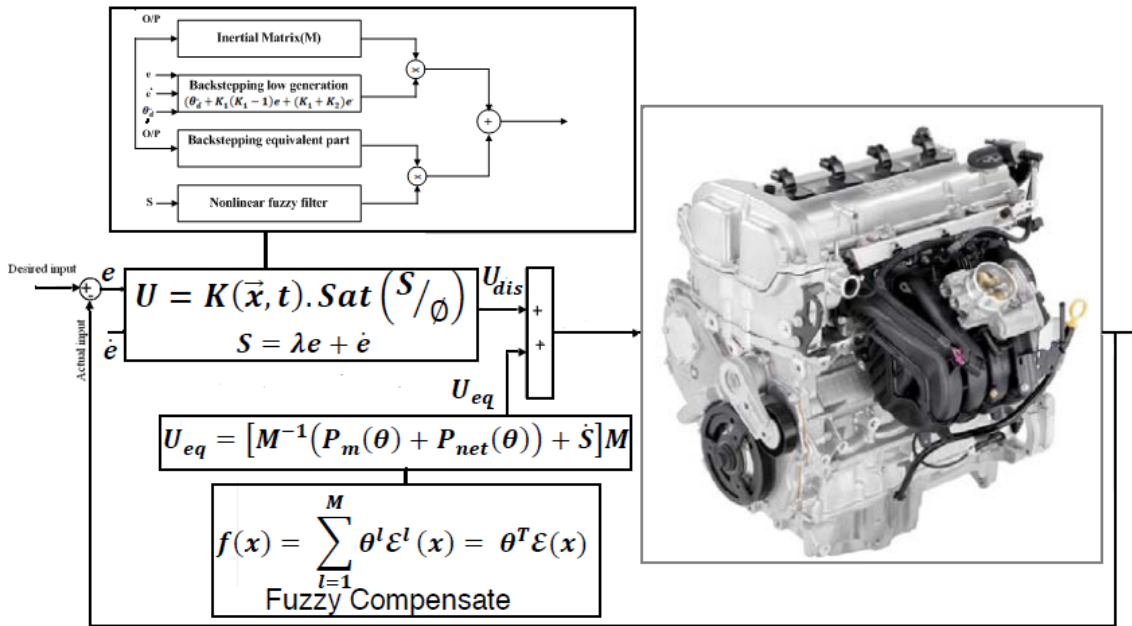


Fig. 7: Chattering free Block diagram of a MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller.

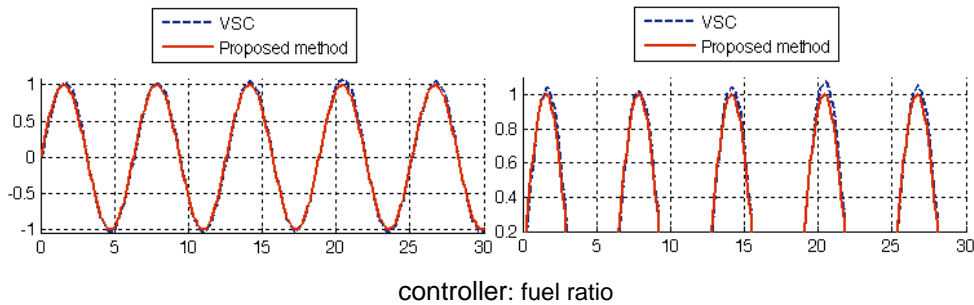


Fig. 8: VSC Vs. MIMO fuzzy backstepping adaptive fuzzy estimator variable structure.

By comparing sinus response, Figure 8, in SMC and MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller, conversely the MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller's overshoot (0%) is lower than VSC's (3%).

**Disturbance Rejection:**

Figure 9 is indicated the power disturbance removal in VSC and MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller. As mentioned before, VSC is one of the most important robust nonlinear controllers. Besides a band limited white noise with predefined of 40% the power of input signal is applied to the sinus VSC and MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller; it found slight oscillations in VSC trajectory responses.

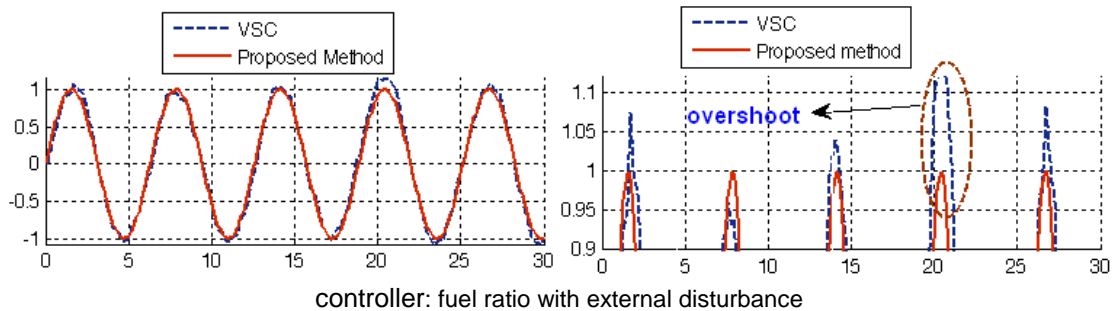


Fig. 9: VSC Vs. MIMO fuzzy backstepping adaptive fuzzy estimator variable structure.

Among above graph, relating to sinus trajectory following with external disturbance, VSC has slightly fluctuations. By comparing overshoot; MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller 's overshoot (0%) is lower than VSC's (12%).

MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller has lower error rate (refer to Table.1), VSC has oscillation tracking which causes chattering phenomenon at the presence of disturbances. Figure 10 is shown steady state and RMS error in VSC and MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller in presence of external disturbance.

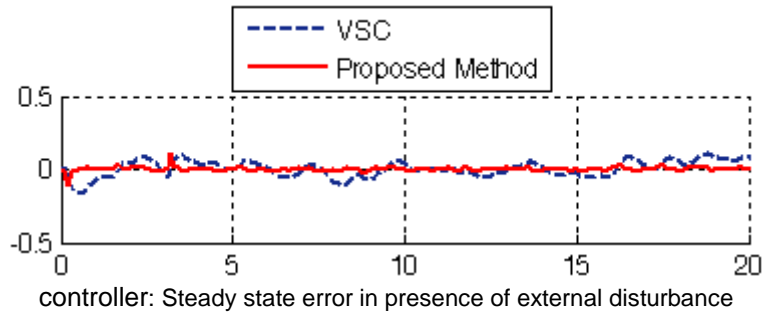


Fig. 10: VSC Vs. MIMO fuzzy backstepping adaptive fuzzy estimator variable structure

In these methods if integration absolute error (IAE) is defined by (67), table 2 is shown comparison between these two methods.

Table 1: RMS Error Rate of Presented controllers.

RMS Error Rate	VSC	Proposed method
Without Noise	1e-3	0.6e-9
With Noise	0.01	0.1e-8

$$IAE = \int_0^{\infty} |e(t)| dt \tag{67}$$

Table 2: Calculate IAE.

Method	VSC	Proposed Method
IAE	442.1	214.8

**Conclusion:**

Refer to the research, a MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller design and application to IC engine has proposed in order to design high performance nonlinear controller in the presence of uncertainties, external disturbances and Lyapunov based. Regarding to the positive points in variable structure controller, fuzzy inference system and adaptive fuzzy backstepping methodology it is found that the adaptation laws derived in the Lyapunov sense. The stability of the closed-loop system is proved mathematically based on the Lyapunov method. The first objective in proposed method is removed the chattering which linear boundary layer method is used to solve this challenge. The second target in this work is compensate the model uncertainty by MIMO fuzzy inference system, in the case of the IC engine, if we define  $k_1$  membership functions for each input variable, the number of fuzzy rules applied for each joint is  $K_1$  which will result in a low computational load. In finally part fuzzy backstepping methodology with minimum rule base is used to online tuning and adjusted the fuzzy variable structure method and eliminates the chattering with minimum computational load. In this case the performance is improved by using the advantages of variable structure, artificial intelligence compensate method and adaptive algorithm while the disadvantages removed by added each method to previous method.

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